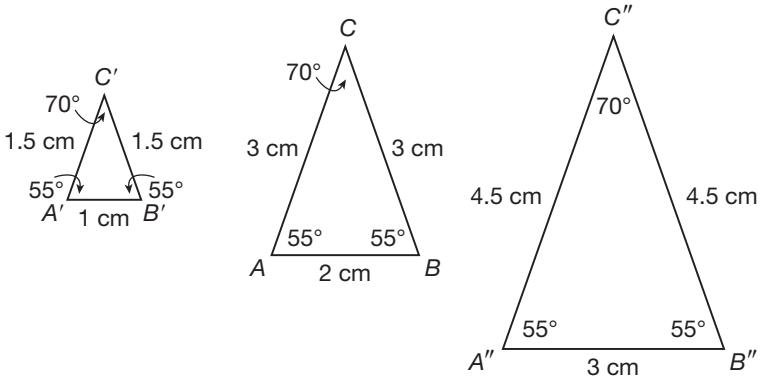
7.5 Understanding Scale

How are the small and large triangles related to $\triangle ABC$?



e.g., All triangles are similar. The corresponding angles are equal. The small triangle is a reduction of $\triangle ABC$ by a scale factor of 0.5. The large triangle is an enlargement of $\triangle ABC$ by a scale factor of 1.5.

Jeb drives a truck. He often needs to read a map. His map of British Columbia shows a scale of 1 cm to 10 km.

scale ratio
a ratio, using the same units, that expresses the scale on a map or drawing

Hint
The scale factor is the number you multiply by. If the scale ratio is $1:x$, then the scale factor is x.

1. What is the **scale ratio** on this map?

$$\frac{1\text{ cm}}{10\text{ km}} = \frac{1\text{ cm}}{\boxed{1\,000\,000}\text{ cm}}$$

The scale ratio is 1:1 000 000.

2. What **scale factor** must Jeb use to convert a distance on the map to the actual distance?

1 cm on the map equals 1 000 000 cm on land. He must multiply distances on the map by a scale factor of 1 000 000.

3. The distance between Calgary and Medicine Hat on the map is 27 cm. What is the actual distance in kilometres?

27 cm × 1 000 000 = 27 000 000 cm

$$27\,000\,000\text{ cm} \times \frac{1\text{ km}}{\boxed{100\,000}\text{ cm}} = 270\text{ km}$$

Example 1

Architects draw scale drawings of homes.
A common scale is $\frac{1}{4}$ in. to 1 ft. The height, length, and width of a home are 6.5 in., 6.5 in., and 10.0 in. on the drawing. What are the actual measurements?

Solution

A. What is the scale ratio on the drawing?

$$\frac{\boxed{0.25}\text{ in.}}{\boxed{1}\text{ ft}} = \frac{\boxed{0.25}\text{ in.}}{\boxed{12}\text{ in.}}$$

Express both terms in the ratio as whole numbers.

$$\frac{\boxed{0.25} \times 4}{\boxed{12} \times 4} = \frac{\boxed{1}}{\boxed{48}}$$

The scale ratio is 1:48.

B. What are the actual dimensions of the home in feet?

H: 48 × 6.5 in. = 312 in.; 312 in. × $\frac{1\text{ ft}}{\boxed{12}\text{ in.}}$ = 26 ft

L: 48 × 6.5 in. = 312 in.; 312 in. × $\frac{1\text{ ft}}{\boxed{12}\text{ in.}}$ = 26 ft

W: 48 × 10.0 in. = 480 in.; 480 in. × $\frac{1\text{ ft}}{\boxed{12}\text{ in.}}$ = 40 ft

The height is $\boxed{26}$ ft. The length is $\boxed{26}$ ft. The width is $\boxed{40}$ ft.

REFLECTING

How could you use a proportion to determine the dimensions?

Example 2

Noreen collects model toy cars. Many of her cars are built using a 1:64 scale. A model of a 1966 convertible is 9.8 cm long and 3.5 cm wide. What does this scale mean? What are the dimensions of the actual car?

Solution

A. The scale means that 1 unit of measurement on the model equals 64 units of the measurement on the actual car. So actual dimensions of the car are 64 times the dimensions of the model.

B. What is the length of the car?

64 × 9.8 cm = 627.2 cm, or 6.272 m

The length of the car is about 6.3 m.

C. What is the width of the car?

64 × 3.5 cm = 224.0 cm, or 2.240 m

The width of the car is about 2.2 m.

REFLECTING

The diameter of the tires on the actual car are 590 mm. What is the diameter of the tires on the model?

Practice

Hint
Ratios are expressed using whole numbers and the same units. See the charts on the back cover for unit conversions.

1. Write each scale as a scale ratio.

a) 1 cm to 1 m

$\frac{1 \text{ cm}}{100 \text{ cm}}$, or 1:100

b) 5 mm to 1 m

$\frac{5 \text{ mm}}{1000 \text{ mm}}$, or 1:200

c) 6 in. to 5 ft

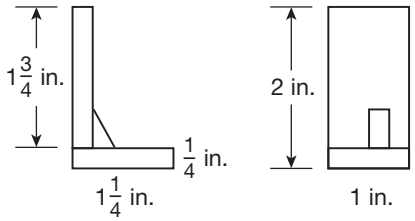
$\frac{6 \text{ in.}}{60 \text{ in.}}$, or 1:10

d) 2 ft to 4 yd

$\frac{2 \text{ ft}}{12 \text{ ft}}$, or 1:6

Hint
Determine and use the scale factor for this problem.

2. Tajana found plans for a bookend in a woodworking magazine. The plans include a scale diagram. The scale ratio is 1:4. What are the length, thickness, and height of the bookend?



e.g., The scale ratio is 1:4, so the scale factor is 4. The dimensions of the actual bookend are 4 times the dimensions of the scale drawing.

Length: $4 \times 1\frac{1}{4}$ in. $= 4 \times \frac{5}{4}$ in., or 5 in.

Width: 4×1 in. $= 4$ in.

Height: 4×2 in. $= 8$ in.

Thickness of base: $4 \times \frac{1}{4}$ in. $= 1$ in.

The length of the bookend is 5 in. The width is 4 in. The height is 8 in. The base is 1 in. thick.

3. Akio drew a building plan. He used a scale of 5 in. on the diagram to represent 6 ft in the building.

a) What is the scale of the plan?

5 in. to 6 ft

b) What is the scale ratio of the plan?

e.g., 5 in.:6 ft $=$ 5 in.:72 in., so the scale ratio is 5:72.